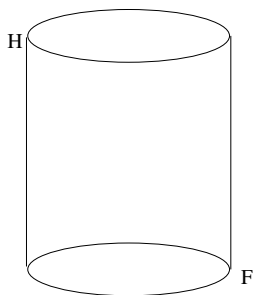


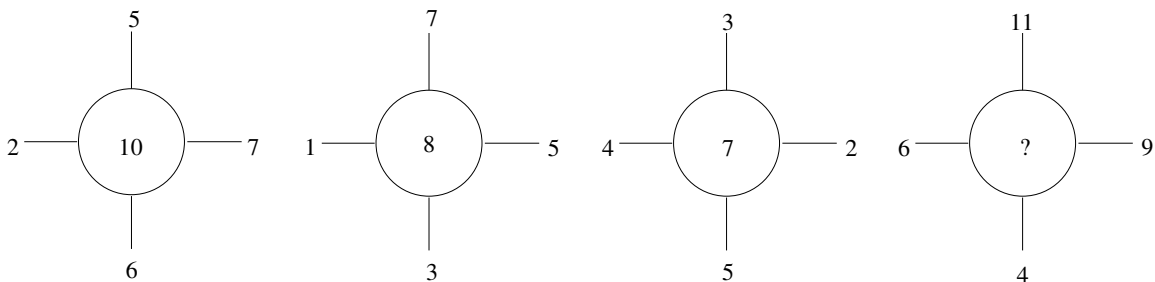
**Part I.** No calculators are allowed. Please do not show your work for Part I. Just write down the final answer on the white paper provided. Each problem is worth 5 points.

1. Compute  $\sin^2 0^\circ + \sin^2 1^\circ + \cdots + \sin^2 89^\circ + \sin^2 90^\circ$ .
2. Rectangle  $ABCD$  has sides  $AB$  and  $BC$  in ratio  $3 : 1$ . If the diagonal  $AC$  has length 5, find the area of the rectangle.
3. If  $x, y, z$  are positive real numbers such that  $xyz = 1$ ,  $x + 1/z = 5$ , and  $y + 1/x = 29$ , find  $z + 1/y$ .
4. The polynomial  $p(x) = x^5 - 3x^4 - 11x^3 + 27x^2 + 10x - 24$  has five real roots. Find the sum of the squares of the roots.
5. A cylindrical cup is four inches high and six inches in circumference. On the top edge of the cup is a drop of honey. Directly opposite, on the bottom edge of the cup, is a fly. What is the minimum distance the fly must walk to reach the honey?



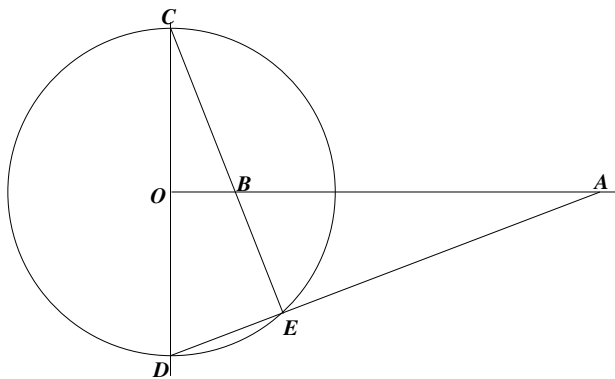
6. There are a number of planes on an island. The fuel tank of each plane, when filled, contains enough fuel to take it halfway around the world. It is possible to transfer fuel from one plane to another during flight, but there is no other source of fuel available. The pilots are told that one plane must fly around the world and that every plane must return to the island. What is the absolute minimum number of planes needed so that one of them can fly around the world?
7. All of the residents of a certain island are either mathematicians or politicians. Everything the mathematicians say is the truth, and everything the politicians say is a lie. One day you run into Alex, Bob, and Carl. Alex says: "All three of us are politicians." Bob says: "Exactly one of us is a mathematician." Determine whether each of Alex, Bob, and Carl are mathematicians or politicians.

8. For each of the four wheels, the numbers around the wheel and the number in the middle of the circle bear a certain relationship to each other. (The same relationship for all four wheels.) What is the missing number?



**Part II.** Please show all work on the yellow paper provided. Each problem is worth 20 points.

9. In the figure below, the circle centered at  $O$  has radius 1 and segment  $CD$  is perpendicular to segment  $OA$ . If the segment  $OA$  has length  $r$ , find the length of segment  $OB$ .



10. A *palindrome number* is an integer that reads the same backwards as forwards (e.g., 2468642). Let  $X$  be the set of all 11-digit positive numbers. An integer is chosen at random from  $X$ . Compute the probability that it is a palindrome number.

11. If eggs are removed from a basket 2, 3, 4, 5, and 6 at a time, there remain, respectively, 1, 2, 3, 4, and 5 eggs. But if the eggs are removed 7 at a time, no eggs remain. What is the least number of eggs that could have been in the basket?