

Part I. No calculators are allowed. Please do not show your work for Part I. Just write down the final answer on the white paper provided. Each problem is worth 5 points.

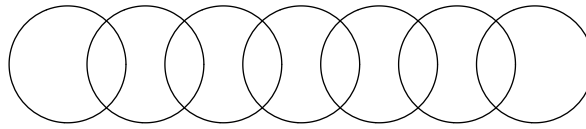
1. The triple (a, b, c) of real numbers has the property that $a + b + c = 5$ and $ab + ac + bc = 5$. What are the possible values of $a^2 + b^2 + c^2$?

2. A rectangular sheet of paper $ABCD$ is three inches in width and four inches in length. The paper is folded so that the two diagonally opposite corners A and C coincide. Determine the length of the crease in the paper.

3. What is the sum of the coefficients of the polynomial obtained by expanding:

$$p(x) = (1 - 3x + 3x^2)^{2005}(1 + 2x - 2x^2)^{2006}?$$

4. A traveller has a straight gold chain with 7 links. She plans to stay at an inn which is willing to accept one link as payment for one night's lodging. She doesn't know when she'll want to leave, so she wants to pay on a daily basis. She'll have to cut some of the links in her chain of course; the inn is perfectly willing to accept cut links as payment. What is the smallest number of links she is forced to cut in order that she is able to pay every night (possibly getting change in the form of links paid earlier) for up to a week? Which link(s) should she break?



5. Given that $13 = 2^2 + 3^2$ and $74 = 5^2 + 7^2$, express $13 \times 74 = 962$ as a sum of two squares.

6. In a certain college of under 5000 total enrollment, a third of the students were freshmen, two-sevenths were sophomores, a fifth were juniors and the rest seniors. The math department offered a popular course in which were registered a fortieth of all the freshmen at the college, a sixteenth of all the sophomores, and a ninth of all the juniors, while the remaining third of the math class were all seniors. How many students were there in the math class?

7. If z is a real number between π and 2π , and $\sin z = -2/3$, what is $81(\sin 2z)^2$?

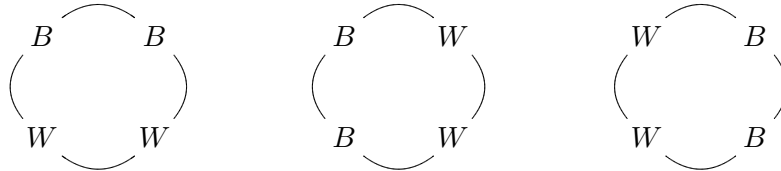
8. Compute $\sum_{i=1}^{99} \frac{200}{i(i+1)}$.

Part II. Please show all work on the yellow paper provided. Each problem is worth 20 points.

9. Let $n > 1$ be a fixed positive integer. There are n coins on a table with each showing tails. A move consists of turning over all but one coin. For which values of n can one have all coins showing heads after a certain number of moves?

10. A villager pays rent to a local farmer who allows him to graze his pet goat on his (the farmer's) field. The goat is tethered to a post by a 21 foot rope, so it grazes in a 21 foot circle. The farmer has decided to build a 14 foot by 7 foot shed in the field using the post as one of the corners of the shed. If the farmer has been charging the villager \$100 per year, by how much should the rent be reduced to reflect the lost grazing area?

11. A group of children has decided to make some necklaces. They have an unlimited supply of round beads in two colors: black and white. Each necklace will have exactly four beads on it, and any combination of colors may be used (that is, repeating colors is allowed). How many different necklaces can the children make? For example, note that the following three necklaces are all the same.



The second is obtained from the first by counterclockwise rotation, while the third is obtained from the second by flipping it over. We would not call these different necklaces, right?