High order numerical method for three-dimensional geophysical flow

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Abstract

The Planetary Geostrophic Equations (PGEs) play a central role in large-scale ocean circulation theory. There is only one dynamic equation for the temperature field and the 3-D velocity field is determined by the planetary geostrophic balance, hydrostatic balance, and the incompressibility condition. The system is reformulated such that all the velocity profiles can be accurately represented as functionals of temperature gradient, by utilizing the special form of the Coriolis parameter. As a result, the PDE system is shown to be well-posed and the corresponding numerical method can be efficiently proposed. The 3-D MAC (marked at cell) scheme, which gives values of physical variables on staggered mesh grid points, are chosen as spatial discretization. The usage of such a staggered grid assures the computed velocity field satisfies the divergence-free property in a discrete level. Furthermore, applying long stencil and compact difference approximations on the grid leads to an efficient fourth order scheme, which is a widely accepted way to improve the accuracy within the limited resolution due to the enormous large scale of the three-dimensional setting. Convergence analysis for the numerical schemes is given, and some computational results of large-scale oceanic circulation are also presented.