

# Asymptotic spatial patterns and entire solutions of semilinear elliptic equations

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## Abstract

In singularly perturbed reaction-diffusion equations on bounded domains, when the diffusion coefficient tends to zero, the behavior of the steady state solutions depends on the qualitative properties of solutions of elliptic equations on the whole space or the half space. The bounded solutions of  $\Delta u + f(u) = 0$  on the whole space or the half space (with certain boundary condition) determine the local asymptotic spatial behavior of solutions to singularly perturbed problems. In this talk we will survey results on entire solutions from the classical Liouville theorem, radially symmetric solutions, to recent development on De Giorgi's conjecture. In these earlier works, the nonexistence of patterns has shown for certain nonlinearities, and the typical patterns found are either radially symmetric or monotone. In the talk, periodic patterns and saddle solutions from the speaker's work will be introduced in details, and related conjectures will also be discussed at the end.