TITLE: Nonstandard ideals from nonstandard dual pairs for $L^1(\omega)$

ABSTRACT: The Banach convolution algebras $l^1(\omega)$ and their continuous counterparts $L^1(\mathbb{R}^+,\omega)$ are much studied, because (when the submultiplicative weight function ω is radical) they are pretty much the prototypic examples of commutative radical Banach algebras. In cases of nice weights ω, the only closed ideals they have are the obvious - or "standard" - ideals . But in the general case, a brilliant but very difficult paper of Marc Thomas shows that nonstandard ideals exist in $l^1(\omega)$. His proof was successfully exported to the continuous case $L^1(\mathbb{R}^+,\omega)$ by Dales and McClure, but remained difficult. We first present a small improvement: a new and easier proof of the existence of nonstandard ideals in $l^1(\omega)$ and $L^1(\mathbb{R}^+,\omega)$. The new proof is based on the idea of a "nonstandard dual pair" which we introduce. We are then able to make a much larger improvement: we find nonstandard ideals in $L^1(\mathbb{R}^+,\omega)$ containing functions whose supports extend all the way down to zero in $\mathbb{R}^+$, thereby solving what has become a notorious problem in the area.