Recent Results on Valence of Certain Harmonic Mappings and Applications to Microlensing

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ABSTRACT

The Fundamental Theorem of Algebra first rigorously proved by Gauss states that each complex polynomial of degree n has precisely n complex roots. In recent years various extensions of this celebrated result have been considered. In this talk we discuss the extension of the FT of algebra to harmonic polynomials of degree n. In particular, a recent theorem of D. Khavinson and G. Swiatek proves that the harmonic polynomial $\bar{z} - p(z), deg\, p = n > 1$ has at most $3n - 2$ roots as was conjectured in the early 90ś by T. Sheil-Small and A. Wilmshurst. Unexpectedly, the proof of the general result involves complex dynamical systems. Moreover, recently, using Levyś theorem on topological polynomials and Thurstonś topological characterization of rational maps L. Geyer was able to show that the result is sharp for all n.

Last year G. Neumann and D. Khavinson showed that the maximal number of zeros of rational harmonic functions $\bar{z} - r(z), deg\, r = n > 1$ is $5n - 5$. It turned out that this result resolved the conjecture by several astrophysists dealing with the estimate on maximal number of images of a star if the light from it is deflected by n co-planar masses. The first non-trivial case of one mass was already studied by A. Einstein in the 30s. Further applications and open problems will be discussed as well.