Some Isoperimetric Problems Arising in the Physics of Thin Structures

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Abstract

Nanophysics is concerned with thin structures such as quantum wires and waveguides, that are on the quantum scale in at least some dimensions. The Schrödinger equation describing an electron in these situations is determined by the shape of the thin structure, and the geometry shows up in the spectrum of the Hamiltonian. I will begin with an explanation of the sorts of Schrödinger equations that arise in the physics of thin structures. Then some sharp inequalities for eigenvalues will be presented, and in some circumstances these are shown to be saturated in the cases of circles or spheres. One result of this kind, obtained recently in joint work with Loss and Exner, follows from a new "isoperimetric" theorem of a classical type, viz., that for $p \leq 2$, the L^p norm of the chord $x(s) - x(s+a)$ between two points separated by arclength a on a closed curve of specified total length is maximized by the circle. This is false for large p. Some non-isoperimetric optimizers and some open questions will also be discussed.