Department of Mathematics and Statistics
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Colloquium

Hamilton cycles and paths in vertex-transitive graphs

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Allen 14

Abstract. A path (cycle) containing every vertex in a graph is called a *Hamilton path* (*Hamilton cycle,* respectively). Hamilton cycles have been studied extensively in graph theory for their own sake, because of connections with the four color problem, and the traveling salesman problem. A graph is called *vertex-transitive* if for any pair of vertices u and v there exists an automorphism mapping u to v. In 1969, Lovasz asked whether every finite connected vertex-transitive graph has a Hamilton path, thus tying together two seemingly unrelated concepts: traversability and symmetry of graphs. With the exception of the complete graph on two vertices, only four connected vertex-transitive graphs that do not have a Hamilton cycle are known to exist. These four graphs are the Petersen graph, the Coxeter graph and the two graphs obtained from them by replacing each vertex by a triangle. The fact that none of these four graphs is a Cayley graph has led to a folklore conjecture that every Cayley graph has a Hamilton cycle. (A Cayley graph is a graph whose automorphism group admits a regular subgroup.) Both of these two problems are still open. However, a considerable amount of partial results are known. In this talk an overview of these results will be introduced. A special emphasis will be given to recent results concerning the existence of Hamilton cycles in cubic Cayley graphs arising from groups having $(2, s, 3)$-presentation.

There will be a reception for Dr. Kutnar in Allen 467 at 3:00 pm.

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