

The characteristic polynomial and determinant are not ad hoc constructions

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Abstract

How do you define the determinant of a matrix? As an alternating sum of products of entries in the matrix (like Jacobi)? Where does that magical formula come from? And what about the characteristic polynomial? Other algebraic structures, like the quaternions, also have notions of determinant and characteristic polynomial. But the definitions one sees in linear algebra don't work for those cases. In fact, the determinant and characteristic polynomial can be defined for any finite-dimensional algebra over a field (e.g., $n \times n$ matrices, the quaternions, a finite-degree field extension). In the case of matrices, one gets the "magical formula" as a consequence.